Physics Qualifying Examination

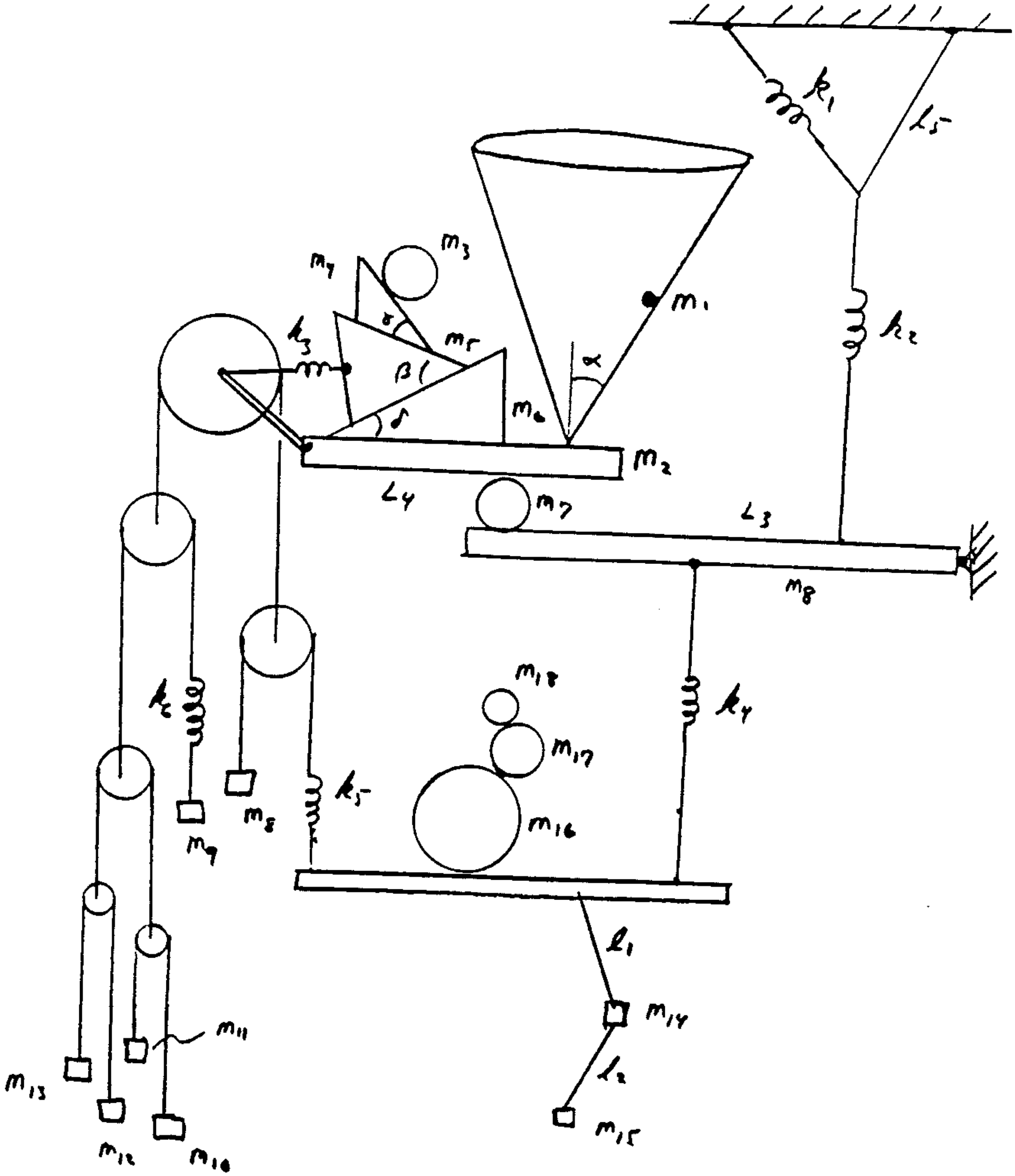
September 19, 1988
8:30 p.m. PSTR 105

CLASSICAL MECHANICS/THERMAL PHYSICS

This is a closed book, closed notes exam.

Write problems 0(optional) through 3 in one Blue Book

Problems 4 - 6 (points noted) in another Blue Book a)



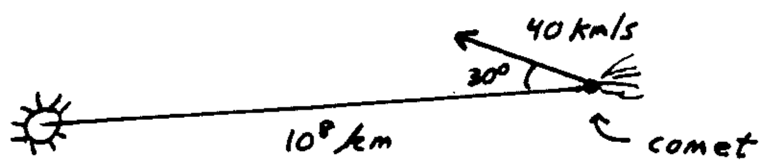
The above system is initially motionless. Find and solve the equations of motion for the system. (This problem is optional.) To simplify this problem you may ignore Coriolis forces associated with the earth's rotation.

1.

a) A rock lies on the ground at a distance R from the center of a merry-go-round which is rotating with angular velocity ω. With respect to coordinates fixed to the merry-go-round the rock moves with uniform speed in a circle, and hence is seen to have an acceleration $\omega^2 r$.
Account for the magnitude and direction of this acceleration in terms of the ficticious forces judged to be acting on the rock. Show the magnitude and direction of all relevant vectors in a clear diagram.
(The equation relating motion in the two frames is:

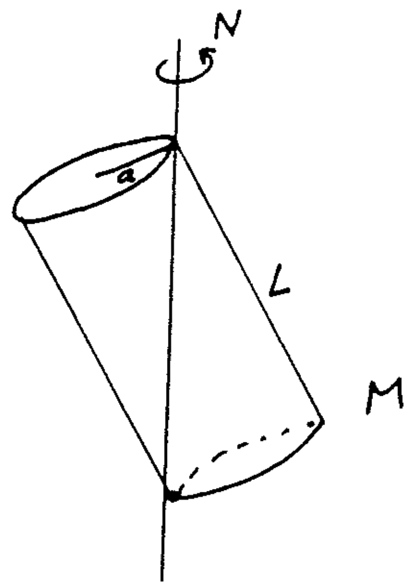
$$\frac{d^2\vec{r}^*}{dt^2} = \frac{d^2\vec{r}}{dt^2} - \vec{\omega}\times(\vec{\omega}\times\vec{r}) - 2\,\vec{\omega}\times\frac{d^*\vec{r}}{dt} - \frac{d\vec{\omega}}{dt}\times\vec{r}$$

b) A comet is observed 10^8 km from the sun, moving at a speed of 40 km/sec in the direction indicated below:



What is the closest that this comet will come to the sun?

2.



A uniform cylinder of radius a, length L, mass M, is rigidly attached to a massless rigid axle as shown. (The axle passes thru azimuthally diametrically opposite points at the two ends of the cylinder, and is constrained to have a fixed direction in space.)

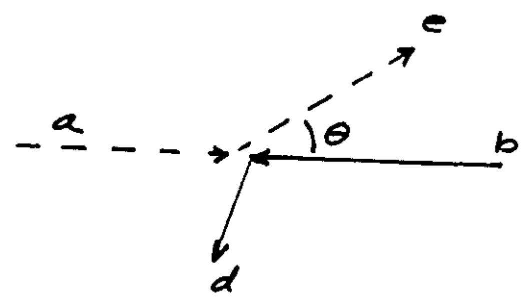
A constant torque N is applied to the axle. The system is initially motionless.

a) Determine the moment of inertial tensor I for the cylinder in a coordinate system aligned with the principal axes of the cylinder.

b) Find the angular velocity of the system at time t.

c) Find the kinetic energy at this time.

3)



A photon "a" with energy a_o makes a head-on collision with a moving particle "b" of mass m and energy b_o. The result is a scattered photon "e" and a particle "d" of mass M. Find the angle θ as a function of the energies of particles a,b, and e. (Let c = 1).

4. a) Apply the general thermodynamic relation $TdS = d\bar{E} + \bar{p}dV$ to a gas of photons. Use $\bar{E} = V\bar{u}$ and $\bar{p} = \frac{1}{3}\bar{u}$, where $\bar{u}(T)$ is the mean energy density, to express dS in terms of dT and dV. Find

$$\left(\frac{\partial S}{\partial T}\right)_V \text{ and } \left(\frac{\partial S}{\partial V}\right)_T \quad . \qquad (2)$$

b) Derive the Stefan-Boltzmann law $\bar{u} \sim T^4$ from the results of part a.

(3)

5. For a system with given mean energy $\bar{E}$ show that the canonical distribution maximes the entropy $S = -k \sum p_i \ln p_i$.

(5)

6. a) Show that in a d-dimensional harmonic crystal the low frequency density of states varies as ω^{d-1}.

(3)

b) Derive from this that the low-temperature specific heat of a d-dimensional crystal varies as T^d.

(4)

c) Show that for a dispersion law $\omega \sim k^s$ (instead of the usual linear $\omega = ck$ relation for phonons) the low-temperature specific heat vanishes as

$$T^{\frac{d}{s}} \quad . \qquad (3)$$

Physics Qualifying Examination

September 19, 1988
2:00 p.m. PSTR 105

QUANTUM MECHANICS

This is a closed book, closed notes exam.
Write problems 1 - 3 in one blue book, 4 & 5 in another blue book

1. Ψ obeys the time dependent Schrodinger equation

$$H\Psi = i\hbar\, \partial\Psi/\partial t$$

where
$$H = \frac{1}{2m}\left[\underset{\sim}{p} + \frac{e}{c}\underset{\sim}{A}(\underset{\sim}{x})\right]^2 - eV(\underset{\sim}{x})$$

$V(\underset{\sim}{x})$, $\underset{\sim}{A}(\underset{\sim}{x})$ are scalar and vector potentials.

a) prove that charge is conserved ie.,

$$\frac{d}{dt}\int \rho\, d\underset{\sim}{x} = 0 \qquad \text{where } \rho = -e|\Psi|^2$$

b) define the current density $\underset{\sim}{j}$ so that the continuity equation $\nabla \cdot \underset{\sim}{j} + \frac{\partial\rho}{\partial t} = 0$ is satisfied.

2. Consider the carbon atom for which Z = 6

a) What is the electronic configuration?

b) What are the possible valves of spin S and orbital angular momentum L for this atom?

c) What are the possibilities for the ground state. (Why must the states $^1P_1$, $^3S_1$, $^3D_1$, $^3D_2$ and $^3D_3$ be rejected?)

d) The ground state is ^3P_o. What rules lead to this identification?

3. A polarized beam of current J_+ contains electrons with spins aligned parallel to a constant magnetic field $\underset{\sim}{B}$ that points in the z-direction. The beam propagates in the x-direction; the wavefunction for a beam electron is of the form

$$\Psi = e^{i(kx-\omega t)} \binom{1}{o}$$

where $\binom{1}{o}$ is the spin function for parallel spin. A monochromatic electromagnetic field of frequency $\hbar\omega = 2\mu_B B$ extends over a length of beam path L_x. A Stern Gerlach analyzer is in the path of the beam beyond the domain of the electromagnetic field. The beam is split into beams containing electrons parallel or anti-parallel to $\underset{\sim}{B}$. If the electron speed is v, what is the current of anti-parallel electrons which are separated from the parallel beam by the S.G. analyzer, ie., what is J_-/J_+.

[The interaction between electrons of the beam and the electromagnetic wave is of the form

$$H' = - 2 H_I \cos\omega t$$

where $$H_I = \mu_B(\underset{\sim}{\sigma} \cdot \underset{\sim}{B}')/2 \quad ,$$

$\mu_B = e\hbar/2\,mc$, $\underset{\sim}{\sigma}$ is the spin operator and $\underset{\sim}{B}'$ is the amplitude of magnetic field of the electromagnetic wave that is assumed to be constant over the distance L_x and the beam radius, and $|\underset{\sim}{B}'| << \underset{\sim}{B}$. According to time dependent perturbation theory the transition probability is

$$P_{(k \to \ell)} = 4 \frac{|\langle k|H_I|\ell\rangle|^2}{\hbar^2(\omega_{k\ell}-\omega)^2} \sin^2\left[\frac{1}{2}\left(\omega_{k\ell}-\omega\right)t\right]$$

where $$\omega_{k\ell} = \left(E_k - E_\ell\right)/\hbar = 2\,\mu_B B_z/\hbar\ ]$$

4. Derive the results of time-independent perturbation theory for the first and second order corrections to the energy of a level, and for the first order correction to the eigenstate.

5. (A) Write down the spin wavefunction for total S, M_s for two electrons. What is the value of $\bar{S}_1 \cdot \bar{S}_2$ in each of these states?

(B) Write down the complete space-spin wave function for 3 electrons in the ground state of a one-dimensional infinite square well potential of length L.